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不完全資訊審計賽局之策略分析

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摘要

本研究以不完全資訊賽局模式來研究管理者與審計人員間之策略互動。該主題過去之分析性研究往往把焦點放在提高審計人員之法律責任是否能給審計人員提高努力之動機從而提升會計資訊之品質。本研究在模式中另外考慮當管理者錯誤報導時將承擔之處罰，並研究其對均衡造成之影響。研究結果發現，若想要達到使審計人員付出高努力之均衡，單靠提高對審計人員之處罰不一定有用，我們必須同時設定管理者與審計人員之適當處罰才能有效地提升審計人員之努力。另外，本研究用了較多之均衡精煉以過濾出同時兼具合理性與穩定性之均衡。這些也是在過往相關研究所未曾用過的。

關鍵詞：審計、賽局、均衡、處罰、精煉

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A Strategic Analysis in an Incomplete Information Auditing Game

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Abstract

This paper uses an incomplete information game-theoretical model to study the interaction between managers and auditors. Previous analytical research about this topic usually focuses mainly on whether raising auditors' legal liability can give auditors incentives to provide more effort and increase the quality of accounting information. We further consider the penalties on managers' misreporting and investigate their effect on the equilibria. Our results show that the equilibrium containing auditors' high effort might not be attained by simply raising the punishments on them. Instead, we have to set the penalties on both parties properly to induce auditors' effort effectively. Besides, this paper uses more equilibrium refinements to find out the equilibria which are both reasonable and stable. This has never been used in the previous research.

Keywords: Auditing; Game; Equilibrium; Penalty; Refinement

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1. Introduction

Accounting is the financial information system that provides relevant financial information about firms to every person who needs such information. This information is communicated through the financial reports prepared by managers. But the information contained in the financial reports does not always fairly present the financial position and operating results of the firm. For example, managers might intentionally misreport to maximize his own benefits. That is, the quality of financial information will indeed be affected by the manager's reporting strategies.

One way to overcome the misreporting problem is to use auditing to determine whether a firm's financial reports present fairly the financial position and operating results of the firm, and to determine whether the financial reporting method used is consistent with generally accepted accounting principles. In order to determine the fairness of the financial report, the auditor examines the firm's internal control system, tests the accuracy of the firm's account balances, and ensures that there are no material errors in the firm's financial reports.

The examination and evaluation of the firm's internal control system and the tests of accuracy of account balances can improve the quality of accounting information. This is because any errors discovered through audit work would be corrected. Besides, the manager's incentive to misreport will be suppressed if he knows the financial reports will be audited, and thus the quality of accounting information will also be improved. The manager might be penalized if the financial report he prepares is qualified by the auditor. Thus the manager's reporting strategy will be affected by the auditing strategy of the auditor. Similarly, the auditor's strategy varies with the manager's strategy, too. Hence there exists an obvious strategic interaction between manager and auditor in this scenario. And it is thus very important to capture this interaction when studying the auditing problems.

It is interesting to know what strategies are the "best" ones for the manager and the auditor respectively in this setting. The traditional single person decision theory is not applicable in this setting because it concerns maximizing one's payoff given all the other players' strategies without modeling the interactions explicitly. On the contrary, the fast-developed game theory characterizes the strategic interactions among two or more parties (e.g., Aumann 1985). Thus, we are motivated to apply game theory to an auditing scenario and try to answer the following questions: How do the manager's and the auditor's strategies affect each other? What is the quality of accounting information under different circumstances? Can the quality of accounting information be improved by raising the penalties on the manager or the auditor if they do not report the truth?

Fellingham and Newman's (1985) paper was the first accounting research using a game-theoretic model to study the strategic interaction between managers and auditors in audit planning and in assessing audit risk. They choose the Nash equilibrium concept, which characterizes a pair of the manager's and the auditor's strategies that can maximize one party's payoff against the other party's strategy, to find all the possible outcomes in their setting. They show that the auditor's strategy depends on the interaction between the accounting control system and the manager's action. After Fellingham and Newman's paper, many research applied game-theoretic models to study the auditors' legal liability problem (for example, Melumad and Thoman 1990; Thoman 1996; Schwartz 1997; Chan and Pae 1998; Hillegeist 1999; Radhakrishnan 1999; Zhang and Thoman 1999; Pae and Yoo 2001; Chan and Wong 2002; Liu and Wang 2006; Lu and Sapra 2009; Laux and Newman 2010). These researches focused mainly on how different liability rules affect the auditors' effort, audit quality or firms' investment decisions. However, most of them did not consider the role of the penalties on the managers in the auditing game. The first important contribution of this paper is we include this important factor in the game-theoretic model and analyze the interaction between auditors' and managers' liabilities. In equilibrium one party has to select the best strategy for himself, conditional on the other party's strategy and the potential penalties on the two parties. For example, if the penalties on the managers' misreporting behavior are extremely severe, the manager will always report truthfully even if the auditor will definitely endorse the financial report.

Kofman and Lawarree (1993, KL hereafter) also considered the effect of punishment on managers in auditing problems. While KL's agency model focuses on the relationship of such punishment and the cost of preventing collusion between managers and auditors, our model emphasizes the importance of setting appropriate punishment to increase the quality of the information in the financial report.

To distinguish the different kinds of quality of accounting information among all the possible outcomes, we classify two kinds of equilibrium. In the "noiseless equilibrium" the financial report reflects the truth perfectly so that the quality of accounting information depends on how effective the audit work is. The harder the auditor works, the higher the quality of accounting information is.

Our result shows that there exists only noiseless equilibrium in which severe penalties for misreporting force the manager to always tell the truth and the auditor does nothing but endorse the manager's financial report. The implication is that if we can really impose very heavy punishment on the manager, he will not misreport no matter what the auditor's strategy is. If this happens, however, we will not need auditing anymore. Thus, auditing is ex ante valuable only if there are some incentives for the manager to misreport.

On the other hand, we find infinitely many noisy equilibria in our model and the quality of accounting information differ a lot in these equilibria. As a matter of fact, it is very common to find many Nash equilibria in a game, and one important area in the game-theoretic research is "refinements" which means eliminating the "bad" Nash equilibria from the equilibrium set. Unfortunately, not many game-theoretic models in previous research have mentioned this problem. Therefore, the second important contribution of this paper is we use several crucial refinements concept to test all the equilibria in our model. We think a good equilibrium should be both reasonable and stable. Reasonableness means the conjecture made by the manager or the auditor should not be incredible. We select sequential equilibrium (Kreps and Wilson 1982) and intuitive criterion (Cho and Kreps 1987) to check the reasonableness of the equilibria. And stability denotes the equilibrium strategy of one party should still be the best response even if we allow the possibility that other parties might make mistakes (in very small probability). We apply the trembling hand perfect equilibrium (Selten 1975) and stable equilibrium (Kohlberg and Mertens 1986) to test if the equilibria we found are stable. We only have to analyze those equilibria passing the above refinements and it simplifies our work a lot.

Our main results are as follows: First, we find many pure and mixed strategy equilibria in our auditing game and there are large differences in the quality of accounting information under various equilibria. Second, only those pure strategy equilibria pass the refinement test. Third, whether we can achieve the most effective equilibrium, which all types of auditors work hard, depends on both parties' penalties. That is, we might not get the most effective equilibrium by raising the auditors' legal liability only. We have to set those penalties properly to improve the quality of accounting information.

The above results can be used to explain some real world scenarios. If we observe that the punishment on the auditors is very severe while there are still some auditors choosing to shirk, then it is possible that the punishment on the manager for being qualified is too severe. Thus we can learn what action should be taken by the policy maker to reach the most effective equilibrium outcome. And it is possible to investigate further about whether it is worthwhile to take specific action by studying its effect on the social welfare.

This paper proceeds as follows: Section 2 describes our auditing game and specifies the possible equilibria. Section 3 presents the refinements of all the equilibria derived in section 2. Section 4 analyzes the effect of different kinds of punishment on the possible equilibrium outcome. Section 5 concludes this paper and proposes some future research directions.

2. Model description and equilibrium specification

We consider a simple one period model in which there are two agents: a manager of the firm and an auditor. The manager runs the firm and hires the auditor to review his financial report.[1] There are two possible "types" of auditors. The difference between different types is the probability to find the errors contained in the financial report.To characterize this, assume (1) the possible value of net income of the firm, $N \in \{N_1, N_2\}$ in which $N_1 < N_2$; (2) the probability to get N_1 for the manager is p, and (3) the probabilities for the type I and II auditors to find out the error, if it exists, in the financial report are s_1 and s_2 respectively, where $s_1 < s_2$.[2]

The actual type of the auditor is determined by nature and is only known to himself. The prior probabilities of the auditor to be type I is $\frac{1}{2}$. That is, without any additional information the manager believes that the auditor he meets belongs to type I with probability $\frac{1}{2}$.

The sequence of the game is:

(1) Nature selects the type of the auditor.

(2) In the end of the period, nature determines the firm's actual net income.

(3) The manager learns the actual result and decides what to present in the financial report.

(4) The auditor chooses the auditing strategy.

(5) The manager and the auditor receive their payoffs.

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[1] There has been some paper which use the principal-agent model to study the auditing problem (e.g., Antle 1982). This approach treats the owner of the firm as a principal to hire the auditor as an agent to monitor the manager. Therefore, owner has to offer the contract that the auditor is willing to accept and serve for him. Contrary to this approach, there is no principal-agent relationship in our manager-auditor model. And we assume that both manager and auditor have to participate the game.

[2] This contains the assumption that the auditor will not commit type I error.

The extensive form game is presented in Figure 1.

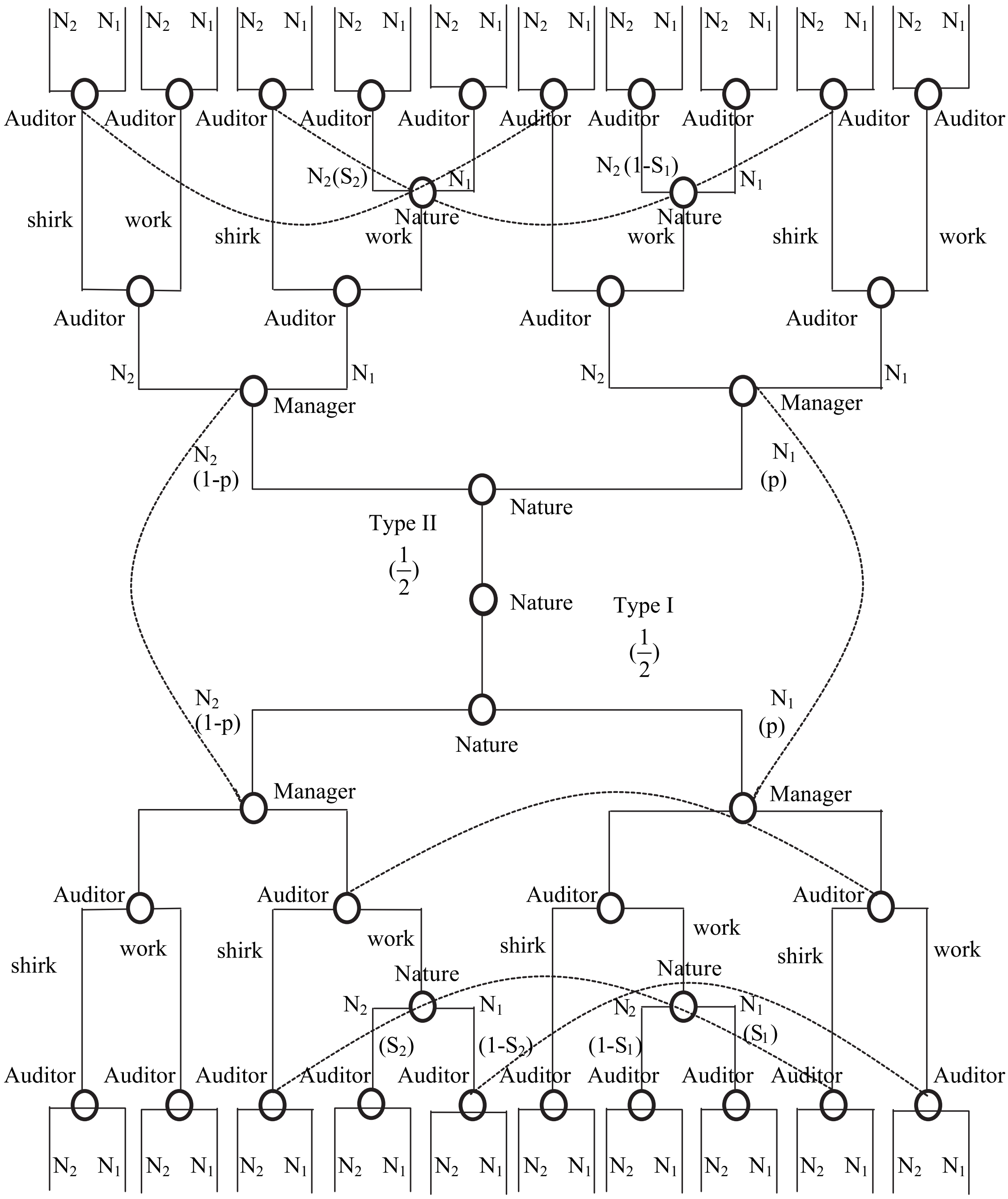

Figure 1 The extensive form of the auditor-manager game

The sets of the available strategies for the manager and the auditor are $\{M_1, M_2\}$ and $\{C_1, C_2, C_3\}$, respectively. On the manager's side, M_1 (M_2) denotes that he reports N_1 (N_2, respectively) in the financial report. On the auditor's side, first he will choose whether to shirk. If he chooses to shirk, he can endorse or qualify manager's financial report. And when the auditor chooses to work hard, he has to decide whether to endorse the manager's financial report according to his findings.[3] He can choose to (a) endorse no matter what he finds, (b) qualify no matter what he finds,[4] (c) report oppositely to his findings, or (d) report truthfully to his findings. Apparently, (a) and (b) is not optimal for the auditor because he will be better off choosing to shirk and endorse (or qualify) to save the effort e. It is also clear that (c) is not optimal, either. This leaves (d) the only possible choice for the auditor after working hard. Consequently, the available strategies for the auditors are as follows: C_1 represents "shirk and endorse manager's financial report", C_2 represents "shirk and qualify manager's financial report", and C_3 represents "work hard and report truthfully". Thus we can define (M_i, M_j), $i, j = 1, 2$ as the manager's strategy in which he chooses M_i when the actual net income is N_1 and M_j when the actual net income is N_2; and define (C_i, C_j), $i, j = 1, 2, 3$ as the auditor's strategy in which he chooses C_i when he receives a low net income financial report and C_j when he receives a high net income financial report.

The manager's payoff is determined partly by the audited net income. For convenience, we assume that the manager can get α, $0 < \alpha < 1$, of the audited net income as his compensation. If the net income stated in the financial report is not consistent with the actual one, two possible punishments might occur to the manager: (1) if the financial report is qualified by the auditor, the punishment is L_a; (2) if the auditor does not qualify the financial report,but the reported net income is not true and is found by the outsiders, with probability r, the punishment is L_m.[5] Audit fees are the auditor's primary payoff. Assume that the auditor can get F_2 if he endorses the manager's financial report with high net income. Otherwise he will get F_1, where $F_1 \le F_2$.[6,7] If there is inconsistency between audited financial report and the actual result which is not found by the auditor, but is found later by the outsiders,

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[3] We assume here that the auditor simultaneously decides whether to work hard and how to report if he chooses to work hard.

[4] We assume that if the auditor qualities the finacial report, the audited net income is not equal to the net income provided by the manager.

[5] L_a includes the cost of adjusting the financial report, the loss of extra bonus, and the possible legal cost. L_m includes the cost due to bad reputation, like the loss of future high compensation jobs, and the indemnity paid to the investors.

[6] This presentation characterizes the possibility of contingent fees. We can simply let the equality hold and omit this problem.

[7] We assume that the low net income company do not have enough money to pay higher audit fee. Thus the auditor can only get F_1 if he endorses the manager's financial report with low net income.

with probability r, the auditor will get punishment L_c.[8] In addition, the auditor will get punishment L_c' if he qualifies the financial report with high net income which is the truth.[9] Besides, if the auditor works hard he has to pay some effort cost, denoted by e.

The payoffs of the manager and the type I auditor in different action combinations are shown in the payoff matrix in Figure 2.[10] Since the manager doesn't know the auditor's type, it is an incomplete information game. Thus we will apply the Bayesian equilibrium concept to find out the possible equilibrium outcome. That is, an equilibrium must consist of a set of strategies and beliefs of the manager and the auditor. Obviously, there might be many possible equilibria in our model. The following lemma will help simplifying the analysis.

(a) the actual net income is N_1

		Manager	
		M_1	M_2
Auditor	C_1	$F_1,\ \alpha N_1$	$F_2 - rL_c,\ \alpha N_2 - rL_m$
	C_2	$F_1 - rL_c,\ \alpha N_2 - L_a$	$F_1,\ \alpha N_1 - L_a$
	C_3	$F_1 - e,\ \alpha N_1$	$s_1F_1 + (1-s_1)(F_2 - rL_c) - e,\ s_1(\alpha N_1 - L_a) + (1-s_1)(\alpha N_2 - rL_m)$

(b) the actual net income is N_2

		Manager	
		M_1	M_2
Auditor	C_1	$F_1 - rL_c,\ \alpha N_1 - rL_m$	$F_2,\ \alpha N_2$
	C_2	$F_1,\ \alpha N_2 - L_a$	$F_1 - L_c',\ \alpha N_1 - L_a$
	C_3	$s_1F_1 + (1-s_1)(F_1 - rL_c) - e,\ s_1(\alpha N_2 - L_a) + (1-s_1)(\alpha N_1 - rL_m)$	$F_2 - e,\ \alpha N_2$

Figure 2 The payoff matrix for the manager and the type I auditor

[8] That is, the expected punishment on the manager (auditor) if there exists inconsistency between audited financial report and actual result is rL_m (rL_c).

[9] The model in this paper is a one-shot game which has the defect in describing the reputation problem. So we use L_c' to represent the reputation loss of the auditor for qualifying a true high net income financial report.

[10] We only present the case of the type I auditor. The other case can be easily derived.

Lemma 1 *If* $\alpha N_1 > \alpha N_2 - rL_m$ *,* M_1 *is the dominant strategy for the manager when the actual net income is* N_1 *.*

Proof: It is straightforward from Figure 2. Q.E.D.

The intuition behind lemma 1 is very straightforward. If the additional payoff the manger can get from misreporting, $\alpha(N_2 - N_1)$, is less than the expected punishment from outsiders, rL_m, he will not misreport even if the auditor will definitely endorse it. It is also clear that, if L_m is very large, the manager will report the truth even without auditing.[11]

One thing people concern very much is the quality of accounting information in the financial report. That is, how accurate the audited financial report can reveal about the firm's actual net income in the equilibrium. Thus, we define:

Definition *Noiseless equilibrium*: In this kind of equilibrium, the net income in the audited financial report is exactly the firm's actual net income. That is, the audited financial report reveals the actual result perfectly and the quality of accounting information is the highest.

Definition *Noisy equilibrium*: In this kind of equilibrium, the net income in the audited financial report may not be the firm's actual net income. That is, the audited financial report cannot reveal the actual result perfectly and the quality of accounting information is not the highest.

Apparently, noiseless equilibrium can only exist when the manager always tells the truth. This is due to the assumption that audit technology is imperfect. If the manager does not always report truthfully the probability of "reported net income is not actual net income" is always positive. From lemma 1, if $\alpha N_1 > \alpha N_2 - rL_m$ M_1 is manager's dominant strategy. In this case the auditor's best response is (C_1,C_1). Thus, we have the following equilibrium:

Equilibrium 1

Manager's strategy: (M_1, M_2)

Auditor's strategy: (C_1, C_1)

Manager's belief: $P(\textit{type I auditor}) = \dfrac{1}{2}$

Auditor's belief: $P(N_1 \mid M_1) = 1$;

$P(N_2 \mid M_2) = 1$.

Condition to sustain the equilibrium: $\alpha N_1 > \alpha N_2 - rL_m$ (1)

[11] This might include nonpecuniary punishment indicated by Kofman and Lawarree (1993).

where $P(N_i \mid M_j)$ denotes the conditional probability of the actual net income is N_i, given the manager chooses M_j.

Equilibrium 1 tells us if the punishment to the manager's misreporting or the probability that the inconsistency is found by the outsiders is large enough, the manager will always tell the truth. In such a case, the existence of the auditor becomes surplus. This result is very intuitive. When the punishment is very rigorous, the manager does not want to lie whether the auditor exists or not. And if the outsiders are smart enough to distinguish most of the inconsistencies in the financial report, they will not need the auditor to provide such information.

The condition for the existence of Equilibrium 1 is strong, and we wonder if the noiseless equilibrium can still sustain even without this condition. That is, if there exists other noiseless equilibrium in which the auditor works hard to force the manager to tell the truth. Unfortunately, the answer is no. It is stated in the following lemma and proposition.

Lemma 2 *"The auditor chooses C_3" cannot be a part of a noiseless equilibrium.*

This result is due to the assumption that audit technology is not perfect. Thus, we cannot exclude the possibility that there is error contained in the audit report, provided the auditor chooses C_3. Unless the manager always report truthfully and the auditor does nothing but endorse the financial report, the noiseless equilibrium cannot be attained. As a matter of fact, given the manager always reports truthfully, the incentive for the auditor to work hard will vanish.

Proposition 1 *Equilibrium 1 is the only possible noiseless equilibrium.*

Proposition 1 indicates that we cannot get a noiseless financial report through the help of auditing. Auditing is valuable only when the manager has, at least a little, probability to misreport. Auditing itself cannot delete the manager's incentive from cheating completely. Again, this strong result is derived under the assumption that even if the auditor works hard, he cannot do the job perfectly. This assumption is reasonable because auditing is based on sampling.

Although the quality of accounting information is the highest in the noiseless equilibrium. It is very hard to get in reality because it is difficult to penalize the manager to remove their incentive to misreport completely. Thus, the study of noisy equilibrium becomes much important. In this kind of equilibrium the manager will not tell the truth all the time.

And we further define: $P(N_1 \mid M_1) = \overline{p}_1$, $P(N_2 \mid M_2) = \overline{p}_2$. Then, $P(N_2 \mid M_1) = 1 - \overline{p}_1$ and $P(N_1 \mid M_2) = 1 - \overline{p}_2$.

Now we can compute the expected payoff of working hard for the auditor. First, if the type I auditor faces M_1, the expected payoff of working hard is:

$$\left[\overline{p}_1+(1-\overline{p}_1)(1-s_1)\right]\left[P^I_{11}\cdot F_1+(1-P^I_{11})(F_1-rL_c)\right]$$
$$+\left[(1-\overline{p}_1)\cdot s_1\right]\left[P^I_{12}\cdot F_1+(1-P^I_{12})(F_1-rL_c)\right]-e$$
$$=F_1-(1-\overline{p}_1)(1-s_1)rL_c-e.$$

And, if the type I auditor faces M_2, the expect payoff of working hard is:

$$\left[(1-\overline{p}_2)\cdot s_1\right]\left[P^I_{12}\cdot F_1+(1-P^I_{21})(F_1-L'_c)\right]$$
$$+\left[(1-\overline{p}_2)(1-s_1)+\overline{p}_2\right]\left[P^I_{22}\cdot F_2+(1-P^I_{22})(F_2-rL_c)\right]-e$$
$$=F_2-(1-\overline{p}_2)\cdot s_1(F_2-F_1)-(1-\overline{p}_2)(1-s_1)rL_c-e.$$

Since there is a unique, noiseless, equilibrium if and only if $\alpha N_1>\alpha N_2-rL_m$, the following analysis about the noisy equilibrium is based entirely on the premise that $\alpha N_1\leq\alpha N_2-rL_m$. That is, if the actual net income is N_1, the manager will not be worse off by reporting N_2, given the auditor chooses C_1.

First, we consider the case that the manager and the auditor use pure strategies only. In this case, when noisy equilibrium occurs, (M_2,M_2) is the only reasonable strategy for the manager.[12] And, we can compute all the expected payoff of the different strategies for the auditor. Let $VC_i(\sigma^c\mid\sigma^m)$ be the expected payoff for type i auditor when he uses the strategy σ^c, given the manager uses the strategy σ^m. We have:

$$VC_I(C_j,C_1\mid M_2,M_2)=VC_{II}(C_j,C_1\mid M_2,M_2)=\overline{p}_2F_2+(1-\overline{p}_2)(F_2-rL_c);$$
$$VC_I(C_j,C_2\mid M_2,M_2)=VC_{II}(C_j,C_2\mid M_2,M_2)=\overline{p}_2(F_1-L'_c)+(1-\overline{p}_2)F_1;$$
$$VC_I(C_j,C_3\mid M_2,M_2)=F_2-(1-\overline{p}_2)s_1(F_2-F_1)-(1-\overline{p}_2)(1-s_1)rL_c-e;$$
$$VC_{II}(C_j,C_3\mid M_2,M_2)=F_2-(1-\overline{p}_2)s_2(F_2-F_1)-(1-\overline{p}_2)(1-s_2)rL_c-e.$$

There are many pure strategy noisy equilibria in this model. One of them is both types of auditors choose to shirk when they get M_2 In this case if one type of auditor chooses C_1 and the other type chooses C_2, it must be:

$$F_2-F_1+\overline{p}_2L'_c>(1-\overline{p}_2)rL_c\quad and\quad F_2-F_1+\overline{p}_2L'_c<(1-\overline{p}_2)rL_c.$$

Obviously they contradict to each other. Thus, if both types of auditors choose C_1 or C_2, they must select the same action in equilibrium. Suppose they both choose C_2 then at least the manager who gets N_1 will deviate to M_1 and upset the equilibrium. The only remaining possibility is both types of auditors choose C_1, and we have the following equilibrium.

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[12] We exclude the equilibia that contains (M_1,M_1) and (M_2,M_1) because it is much unreasonable than other equilibria.

<u>Equilibrium 2</u>

Manager's strategy: chooses (M_2, M_2)

Auditor's strategy: type I chooses (C_1, C_1);

type II chooses (C_1, C_1).

Manager's belief: P (type I auditor) $= \dfrac{1}{2}$

Auditor's belief: $P(N_2 \mid M_2) = \overline{p}_2$;

$P(N_1 \mid M_1) = 1$.[13]

Conditions:

$F_2 - F_1 > (1 - \overline{p}_2) r L_c - \overline{p}_2 L_c'$ (2a)

$s_1 (1 - \overline{p}_2) r L_c - (1 - \overline{p}_2) s_1 (F_2 - F_1) < e$ (2b)

$s_2 (1 - \overline{p}_2) r L_c - (1 - \overline{p}_2) s_2 (F_2 - F_1) < e$ (2c)

$\alpha N_1 < \alpha N_2 - r L_m$ (2d)

When the manager chooses (M_2,M_2), (2a) denotes both types of auditors prefer C_1 to C_2; (2b) and (2c) denote type I and II auditors prefer C_1 to C_3, respectively.

Next we consider the noisy equilibrium in which at least one type of auditor chooses C_3. We further assume $rL_c > F_2 - F_1$. Under this assumption if it is not optimal for type II auditor to choose C_3, it cannot be optimal for type I auditor, either. Thus, we can derive the following equilibria:

<u>Equilibrium 3</u>

Manager's strategy: always chooses (M_2, M_2)

Auditor's strategy: chooses (C_1, C_3)

Manager's belief: P (type I auditor) $= \dfrac{1}{2}$

Auditor's belief: $P(N_1 \mid M_2) = \overline{p}_2$;

$P(N_1 \mid M_1) = 1$.

Conditions:

$F_2 - F_1 > (1 - \overline{p}_2) r L_c - \overline{p}_2 L_c'$ (3a)

$s_1 (1 - \overline{p}_2) r L_c - (1 - \overline{p}_2) s_1 (F_2 - F_1) > e$ (3b)

$s_2 (1 - \overline{p}_2) r L_c - (1 - \overline{p}_2) s_2 (F_2 - F_1) > e$ (3c)

$\overline{s}(\alpha N_1 - L_a) + (1 - \overline{s})(\alpha N_2 - r L_m) > \alpha N_1$ (3d)

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[13] As we have mentioned in the previous note, we exclude (M_1, M_1) and (M_2, M_1) in any equilibrium. Therefore the auditor's belief $P(N_1 \mid M_1)$ must be equal to 1.

$$\alpha N_1 < \alpha N_2 - rL_m \tag{3e}$$

where $\overline{s} = \frac{1}{2}s_1 + \frac{1}{2}s_2$.

When the manager chooses (M_2,M_2), (3a) denotes both types of auditors prefer C_1 to C_2; (3b) and (3c) denote both type I and II auditors prefer C_3 to C_1 respectively. And when both types of auditors choose (C_1,C_3), (3d) denotes the manager prefers M_2 to M_1.

Equilibrium 4

Manager's strategy: chooses (M_2, M_2)

Auditor's strategy: type I chooses (C_1, C_1)

type II chooses (C_1, C_3)

Manager's belief: P (type I auditor) $= \frac{1}{2}$

Auditor's belief: $P(N_1 \mid M_2) = \overline{p}_2$;

$P(N_1 \mid M_1) = 1$.

Conditions:

$$F_2 - F_1 + \overline{p}_2 \cdot L_c' > (1 - \overline{p}_2) rL_c \tag{4a}$$

$$s_1(1 - \overline{p}_2) rL_c - (1 - \overline{p}_2) s_1 (F_2 - F_1) < e \tag{4b}$$

$$s_2(1 - \overline{p}_2) rL_c - (1 - \overline{p}_2) s_2 (F_2 - F_1) > e \tag{4c}$$

$$(1 - \overline{p}_2)(1 - s_2) rL_c < (F_2 - F_1)[1 - (1 - \overline{p}_2) s_2] + \overline{p}_2 L_c' - e \tag{4d}$$

$$\frac{1}{2} s_2 (\alpha N_1 - L_a) + \left[\frac{1}{2} + \frac{1}{2}(1 - s_2)\right](\alpha N_2 - rL_m) > \alpha N_1 \tag{4e}$$

$$\alpha N_1 < \alpha N_2 - rL_m \tag{4f}$$

When the manager chooses (M_2,M_2),(4a) and (4b) denote the type I auditor prefers C_1 to C_2 and C_3 respectively, (4c) and (4d) denote the type II auditor prefers C_3 to C_1 and C_2 respectively. When type I and II auditors choose (C_1,C_1) and (C_1,C_3) respectively, (4e) denotes the manager prefers M_2 to M_1.

Equilibrium 5

Manager's strategy: chooses (M_2, M_2)

Auditor's strategy: type I chooses (C_1, C_1);

type II chooses (C_1, C_3).

Manager's belief: P (type I auditor) $= \frac{1}{2}$

Auditor's belief: $P(N_1 \mid M_2) = \overline{p}_2$;

$P(N_1 \mid M_1) = 1$.

Conditions:

$$F_2 - F_1 + \overline{p}_2 L_c' < (1-\overline{p}_2) r L_c \quad (5a)$$

$$(1-\overline{p}_2)(1-s_1) r L_c > (F_2 - F_1)\left[1-(1-\overline{p}_2)s_1\right] + \overline{p}_2 L_c' - e \quad (5b)$$

$$s_2(1-\overline{p}_2) r L_c - (1-\overline{p}_2) s_2 (F_2 - F_1) > e \quad (5c)$$

$$(1-\overline{p}_2)(1-s_2) r L_c > (F_2 - F_1)\left[1-(1-\overline{p}_2)s_2\right] + \overline{p}_2 L_c' - e \quad (5d)$$

$$\left[\frac{1}{2} + \frac{1}{2} s_2\right](\alpha N_2 - L_a) + \frac{1}{2}(1-s_2)(\alpha N_2 - r L_m) > \alpha N_1 \quad (5e)$$

$$\alpha N_1 < \alpha N_2 - r L_m \quad (5f)$$

When the manager chooses (M_2,M_2), (5a) and (5b) denote the type I auditor prefers C_2 to C_1 and C_3 respectively; (5c) and (5d) denote the type II auditor prefers C_3 to C_1 and C_2 respectively. When type I and II auditors choose (C_1,C_2) and (C_1,C_3) respectively, (5e) denotes the manager prefers M_2 to M_1.

Remark We can use the concept of "effective auditing equilibrium" introduced by Melumed and Thoman (1990) to study the property of Equilibrium 2 to Equilibrium 5. They define it as an equilibrium in which the auditor works hard and reports his findings truthfully. Among the four noisy pure strategy equilibria, we can easily see that Equilibrium 2 is the least effective one. In that equilibrium the auditor does nothing but endorse the manager's financial report. Thus auditing is totally valueless if this equilibrium occurs. In contrast, Equilibrium 3 is most effective because both types of auditors will work hard when they receive the financial report with high net income. The accuracy of financial report is the highest in this equilibrium, excluding the noiseless one. The effectiveness of auditing in the remaining two equilibria are between Equilibrium 2 and Equilibrium 3 because only the type II auditor will work hard in these two cases.

The equilibria we have studied so far are all pure strategy equilibria. Now we turn to mixed strategy equilibria which mean that the manager and/or the auditor choose mixed strategy in the equilibrium. The general form of their mixed strategy is as follows:

For the manager:

σ^m : chooses (M_i,M_j) with probability t_{ij}, where $0 \le t_{ij} \le 1$ and $\sum_i \sum_j t_{ij} = 1$

$i, j = 1,2$.

And for the auditor:

σ^c : type I auditor chooses (C_i,C_j) with probability u_{ij} ;

type II auditor chooses (C_i,C_j) with probability v_{ij} ;

where $0 \le u_{ij} \le 1$, $0 \le v_{ij} \le 1$, $\sum_i \sum_j u_{ij} = 1$, and $\sum_i \sum_j v_{ij} = 1$, $i, j = 1,2,3$.

Now we define the possible mixed strategy equilibrium as:

Mixed equilibria

Strategies: the strategies profile $(\sigma^{\mathrm{m}}, \sigma^{\mathrm{c}})$ as described before where $(t_{11}, \cdots, t_{22})$, $(u_{11}, \cdots, u_{33})$ and $(v_{11}, \cdots, v_{33})$ are not all unit vectors.

Manager's belief: P (type I auditor) = $\dfrac{1}{2}$

Auditor's belief: $P(N_1 \mid M_2) = \overline{p}_2$;
$P(N_1 \mid M_1) = 1$.

Conditions:

$$t_{ij} = 0 \text{ if and only if } (M_i, M_j) \notin \arg \max_{(M_i, M_j)'} V\,M((M_i, M_j)' \mid \sigma^c) \quad i,j=1,2,3 \tag{6a}$$

$$u_{ij} = 0 \text{ if and only if } (C_i, C_j) \notin \arg \max_{(C_i, C_j)'} V\,C_{\mathrm{I}}((C_i, C_j)' \mid \sigma^m) \quad i,j=1,2,3 \tag{6b}$$

$$v_{ij} = 0 \text{ if and only if } (C_i, C_j) \notin \arg \max_{(C_i, C_j)'} V\,C_{\mathrm{II}}(\,(C_i, C_j)' \mid \sigma^m) \quad i,j=1,2,3 \tag{6c}$$

where $VM((M_i, M_j)' \mid \sigma^c)$ is the expected payoff for the manager when he chooses $(M_i, M_j)'$ given the auditor chooses σ^c, and $V\,C_{\mathrm{I}}((C_i, C_j)' \mid \sigma^m)$ ($V\,C_{\mathrm{II}}(\,(C_i, C_j)' \mid \sigma^m)$) is the expected payoff for the type I (type II) auditor when he chooses $(C_i, C_j)'$ given the manager choose σ^m.

Apparently, there are infinitely many t_{ij} and u_{ij} or v_{ij} combinations can sustain the mixed equilibria.

3. Refinements of equilibria

Those equilibria derived in section 3 are all Nash equilibria. Now we want to find out if these equilibria are the obvious ways to play by the manager and the auditor. Hence, some refinements on these equilibria are needed.

The refinements we will do in this section can be divided into two parts. First we will check the 'reasonableness' of these equilibria. Then we will test the stability of them. For the first part, reasonableness means the conjecture made by the manager or the auditor should not be incredible. We select the concept of sequential equilibrium (Kreps and Wilson 1982) and intuitive criterion (Cho and Kreps 1987) to check if the equilibria mentioned in section 2 are reasonable.[14]

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[14] Few previous game-theoretical researches checked the reasonableness of equilibria. Zhang and Thomas (1999) used "divinity", proposed by Banks and Sobel (1987), to investigate the beliefs on the off-the-equilibrium path. Melumad and Thoman (1990) introduced the "calculated belief" which is similar to divinity.

The process of examining "one" equilibrium is sequential can be applied to "all" the equilibria in this game. First we select one of them to test, this is stated in the following proposition:

Proposition 2 *Equilibrium 3 is a sequential equilibrium.*[15]

Following the same way of proof of proposition 2, we can also show that all the other equilibria presented in section 3 are sequential too. So we have the following corollary.

Corollary 1 *Equilibrium 1 to Equilibrium 5 and Mixed equilibria are all reasonable in the sense of sequential equilibrium.*

Corollary 1 says that Equilibrium 1 to Equilibrium 5 and Mixed equilibria are all reasonable at least on the equilibrium path. Next we want to check if the beliefs of those equilibria are still reasonable on the off-the-equilibrium-path. This is done by applying "intuitive criterion" presented by Cho and Kreps (1987). The result is the next proposition.

Proposition 3 *Equilibrium 1 to Equilibrium 5 and Mixed equilibria all satisfy intuitive criterion.*

Proof:

Equilibrium 1 and part of Mixed equilibria do not have any off-the-equilibrium-path. Hence, intuitive criterion is satisfied trivially. For Equilibrium 2 to Equilibrium 5 and some of Mixed equilibria, off-the-equilibrium-path occurs when the manager chooses M_1. In these cases what conjecture should the auditor have? In Equilibrium 2 to Equilibrium 4, the equilibrium payoff of the manager who gets net income N_2 is αN_2. And the highest possible payoff he can get from deviating to M_1 is only $\alpha N_2 - L_a$ or $\alpha N_1 - rL_m$. Thus, when the auditor faces M_1, he should have the belief $P(N_2 \mid M_1) = 0$ which is exactly the belief stated in Equilibrium 2 to Equilibrium 4. In Equilibrium 5 and Mixed equilibria, M_1 is not equilibrium dominated at least for the manager who gets N_1. Hence, the beliefs in these equilibria also pass the intuitive criterion, too. This completes the proof of this proposition. Q.E.D.

After checking the reasonableness of these equilibria, we want to know further about their "stability" which is not yet considered in the previous research. The reason we want to do this work is as follows: In the real world it is possible, although the probability may be small, that the manager (or auditor) makes some

[15] The proof of propositions 2, 4, 5 and 6 are in the appendix.

mistakes in selecting their strategies. For example, the manager may misreport his net income in the financial report unintentionally. If the auditor cares about the possibility of this situation, will he still choose the optimal strategy (work hard, for example) stated in the equilibrium? If the answer is no, this equilibrium may not be "stable". We will check the stability of these equilibria subsequently.

First we want to apply the "trembling-hand perfection" introduced by Selten (1975) to test the stability of those equilibria. A strategy profile of the manager and the auditor (σ^m, σ^c) is trembling-hand perfect if we can find a sequence of real numbers $\{\varepsilon_k\}_{k=1}^{\infty}$ and sequences of the manager's and the auditor's totally mixed strategies $\{\sigma_k^m\}_{k=1}^{\infty}$ and $\{\sigma_k^c\}_{k=1}^{\infty}$ which satisfy the following conditions (Van Damme 1991):

(i) $\varepsilon_k > 0$ and ε_k converges to zero.

(ii) (σ_k^m, σ_k^c) is an ε_k-perfect equilibrium.

(iii) (σ_k^m, σ_k^c) converges to (σ^m, σ^c).

And we will check if Equilibrium 1 to Equilibrium 5 and Mixed equilibria satisfy the above conditions. The results are in propositions 4 and 5.

Proposition 4 *Equilibrium 1 is a trembling-hand perfect equilibrium.*

Following the similar way of the proof of proposition 4, we can also show that Equilibrium 2 to Equilibrium 5 are also trembling-hand perfect. Notice that these equilibria are all pure strategy equilibria. But these conclusions cannot be extended to Mixed equilibria. The result is stated in the following proposition.

Proposition 5 *Mixed equilibria are not all trembling-hand perfect, especially when* $\alpha N_1 = \alpha N_2 - rL_m$.

In our auditing game, a trembling-hand perfect equilibrium means that: if there's a little probability for the manager (or auditor) to make some kind of mistake, the equilibrium in the perturbed game will converge to the equilibrium of the original game. But the question is: For what reason must the manager (or auditor) believe that the other party will err in that way? If any other mistake occurs, can the equilibrium we concern still be sustained? Hence, we need some stronger refinement to make sure further if those equilibria we discussed are still stable under any kind of tremble. To do this, we apply the "stable equilibria" concept, introduced by Kohlberg and Mertens (1986), on the equilibria we studied. According to Kohlberg and Mertens, the requirements for strategic stability are:

(i) Existence: Every game has a solution.

(ii) Invariance: Two games with the same reduced normal form have the same solutions.

(iii) Sequential rationality: Every solution contains a sequential equilibrium.

(iv) Admissibility: Every element of a solution is undominated.

(v) Elimination of dominated strategies: A solution contains a solution of a game obtained by eliminating a dominated strategy.

(vi) Elimination of non-best replies: A solution contains a solution of a game obtained by eliminating a strategy that is not a best response against any element of the solution.

(vii) Connectedness: A solution is connected.

Kohlberg and Mertens then define "stable equilibria" as (p. 1027): A set of equilibrium is stable in a game *G* if it is minimal with respect to the following property (property *S*, hereafter): *S* is a closed set of Nash equilibria of *G* satisfying: for any $\varepsilon > 0$ there exists some $\delta_0 > 0$ such that for any completely mixed strategy vector $\sigma_1, \cdots, \sigma_n$ (n players) and for any $\delta_1, \cdots, \delta_n$ ($0 < \delta_i < \delta_0$), the perturbed game where every strategy s of player i is replaced by $(1-\delta_i)s + \delta_i \sigma_i$ has an equilibrium ε-close to S.[16]

In the next proposition we will see that there exist stable equilibria in our model.

<u>Proposition 6</u> *Equilibrium 1 is stable in the sense of Kohlberg and Mertens.*

Proposition 6 tells us that if $\alpha N_1 > \alpha N_2 - rL_m$, Equilibrium 1 can always be sustained under any kind of tremble. That is, the manager always tells the truth and the auditor always submits to the manager are always the best response for them no matter what kind of mistake may happen. And we can also show that Equilibrium 2 to Equilibrium 5 are all stable too. Thus, we have the following corollary.

<u>Corollary 2</u> *All the pure strategy equilibria in our auditing game are stable equilibria.*

We have mentioned before that the concept of stable equilibrium is much stronger than trembling-hand perfection. As Fudenberg and Tirole (1991, p. 444) pointed out "The key difference between trembling-hand perfection … and stability is that perfection requires only that there exists a single sequence of perturbed games whose equilibria converges to σ, but a stable set must contain a limit point of the equilibria for every perturbed game". Using this argument and proposition 5 we immediately know that some of Mixed equilibria are not stable. And the fact is even

[16] This means for any small number $\varepsilon > 0$, there exists an equilibrium in the perturbed game within ε-distance of the equilibrium set of the original game.

stronger which is stated in proposition 7.

Proposition 7 *All of Mixed equilibria are not stable.*

Proof:

Let (σ_k^m, σ_k^c) donate the perturbed strategy profile of any mixed strategy profile. Then it must be that $VM(M_1 \mid \sigma_k^c) = VM(M_2 \mid \sigma_k^c)$ or there exists some type i auditor for which $VC_i(C_a \mid \sigma_k^m) = VC_i(C_b \mid \sigma_k^m)$, $a, b = 1,2,3$ and $a \neq b$. And we can always find some kind of tremble to break the above equality and make one of the strategies dominated. Thus, (σ_k^m, σ_k^c) is not a stable equilibrium. Q.E.D.

The results of proposition 7 help us simplifying the analysis a lot. Notice that there are infinitely many mixed strategy equilibria, but proposition 7 says they are all not stable. So we can neglect them if we only care about the equilibrium that can sustain any kind of mistakes made by the manager or the auditor.

Remark The result in this section indicates that only the pure strategy equilibria are meaningful. And in most cases the manager will only present the high net income in the equilibrium. As we showed before that the manager is willing to present N_1, if it is the actual result, only when the punishment for misreporting is quite large. Put it in another way, if the manager's ability is so low that he can only get N_1, then even little punishment can force him to tell the truth. We can see this is an implication of "lemon problem" introduced by Akerlof (1970).

4. Further analysis

In this section we will discuss some implications about the equilibria which pass the refinements in the previous section. From both regulatory and professional view, one important question is if we can raise the auditors' incentives to work hard through simply increasing the punishment on auditors for misreporting. In the following analysis, we will see this is not always true. This conclusion is derived by analyzing how the penalties on managers and auditors will affect the set of possible equilibria respectively.

Let's first study how the two different punishments on the manager, L_a and L_m can affect the occurrence of equilibria. To see this more clearly, we rewrite the equilibrium conditions regarding to the manager's action of Equilibrium 1 to Equilibrium 5 below:

Equilibrium 1: $$L_m > \frac{1}{r}(\alpha N_2 - \alpha N_1) \qquad (1')$$

Equilibrium 2: $$L_m < \frac{1}{r}(\alpha N_2 - \alpha N_1) \qquad (2')$$

Equilibrium 3: $(1-\bar{s})rL_m + \bar{s}L_a < (1-\bar{s})(\alpha N_2 - \alpha N_1)$ (3c')

$$L_m < \frac{1}{r}(\alpha N_2 - \alpha N_1) \quad \text{(3d')}$$

Equilibrium 4: $\left[\frac{1}{2}+\frac{1}{2}(1-s_2)\right]rL_m + \frac{1}{2}s_2L_a < \left[\frac{1}{2}+\frac{1}{2}(1-s_2)\right](\alpha N_2 - \alpha N_1)$ (4e')

$$L_m < \frac{1}{r}(\alpha N_2 - \alpha N_1) \quad \text{(4f')}$$

Equilibrium 5: $\frac{1}{2}(1-s_2)rL_m + \frac{1}{2}(1+s_2)L_a < \frac{1}{2}(1-s_2)(\alpha N_2 - \alpha N_1)$ *(5e')*

$$L_m < \frac{1}{r}(\alpha N_2 - \alpha N_1) \quad \text{(5f')}$$

From (1') Equilibrium 1 occurs only when $L_m > \frac{1}{r}(\alpha N_2 - \alpha N_1)$. From (2'), (3d'), (4f') and (5f') when $L_m < \frac{1}{r}(\alpha N_2 - \alpha N_1)$, Equilibrium 2 to Equilibrium 5 are all possible. (3c'), (4e') and (5e') are all linear inequalities involving two parameters: L_a and L_m. Taking them as equalities and calculate the intercepts on both axes. The L_m-axis intercepts are all found to be $\frac{1}{r}(\alpha N_2 - \alpha N_1)$, and the L_a-axis intercepts for (3c'), (4e') and (5e') are $\frac{2-s_1-s_2}{s_1+s_2}\times(\alpha N_2 - \alpha N_1)$, $\frac{2-s_2}{s_2}\times(\alpha N_2 - \alpha N_1)$ and $\frac{1-s_2}{1+s_2}\times(\alpha N_2 - \alpha N_1)$ respectively. Now we can present the possible equilibrium regions in Figure 3.

Figure 3 characterizes how different value of L_m and L_a can affect the equilibrium outcome. When L_m is small, L_a plays the key role. If L_a is very large (region II), the manager dare to always report the high net income only when the auditor always endorses his report. Thus, only Equilibrium 2 can be sustained. If L_a is smaller (region IV) Equilibrium 2 still exists, and the manager is also willing to always chooses M_2 if type II auditor chooses C_3 and type I auditor chooses C_1. That is, Equilibrium 4 can be sustained too. If L_a falls in region III, Equilibrium 2 and Equilibrium 4 can still be sustained, and the manager is still willing to always chooses M_2 even when both types of auditors work hard. This makes Equilibrium 3 sustainable. And if L_a is very small (in region V) the manager won't care much about being qualified, thus Equilibrium 2 to Equilibrium 5 can all be sustained.

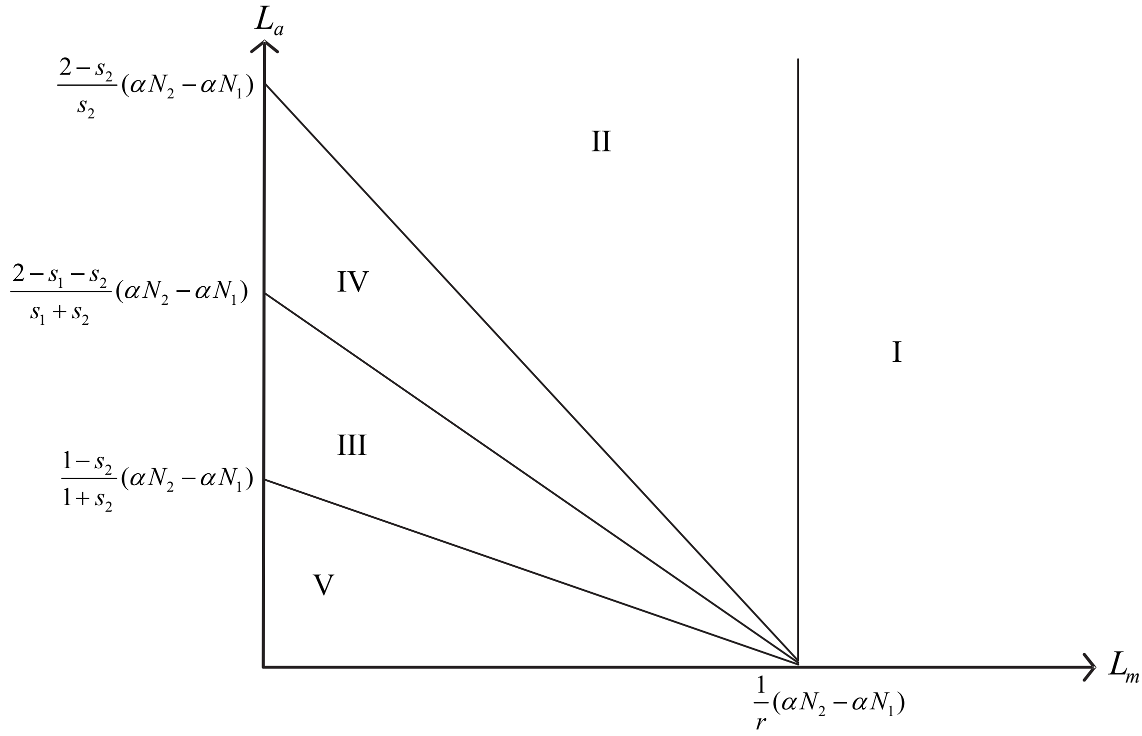

Figure 3 The equilibrium regions on different L_a and L_m

When L_m exceeds $\frac{1}{r}(\alpha N_2-\alpha N_1)$, then as we mentioned before only noiseless equilibrium (Equilibrium 1) exists. If L_m is not that large but close to $\frac{1}{r}(\alpha N_2-\alpha N_1)$ and L_a is also not very large, the equilibrium outcome will be very sensitive to L_a. In this case even a very large L_a will make Equilibrium 2 to be the only possible type of equilibrium. The closer L_m is to (but less than) $\frac{1}{r}(\alpha N_2-\alpha N_1)$, the more sensitive the equilibrium outcome is to L_a.

Next we study what is the effect of the punishment on the auditor, L_c, on the equilibrium outcome. For the ease of analyzing we set $F_1 = F_2$. That is, we assume that the fee is not contingent on outcome. Similarly, we rewrite the equilibrium conditions with respect to the auditor's action of Equilibrium 2 to Equilibrium 5 as follows:

Equilibrium 2: $\overline{p}_2 L_c' > (1-\overline{p}_2) r L_c$ $\quad (2a')$

$$L_c < \frac{e}{r(1-\overline{p}_2)s_1} \quad (2b')$$

$$L_c < \frac{e}{r(1-\overline{p}_2)s_2} \tag{2c'}$$

Equilibrium 3: $$L_c > \frac{e}{r(1-\overline{p}_2)s_1} \tag{3a'}$$

$$L_c < \frac{\overline{p}_2 L_c' - e}{r(1-\overline{p}_2)(1-s_1)} \tag{3b'}$$

Equilibrium 4: $$\overline{p}_2 L_c' > (1-\overline{p}_2) r L_c \tag{4a'}$$

$$L_c < \frac{e}{r(1-\overline{p}_2)s_1} \tag{4b'}$$

$$L_c > \frac{e}{r(1-\overline{p}_2)s_2} \tag{4c'}$$

$$L_c < \frac{\overline{p}_2 L_c' - e}{r(1-\overline{p}_2)(1-s_2)} \tag{4d'}$$

Equilibrium 5: $$\overline{p}_2 L_c' < (1-\overline{p}_2) r L_c \tag{5a'}$$

$$L_c > \frac{\overline{p}_2 L_c' - e}{r(1-\overline{p}_2)(1-s_1)} \tag{5b'}$$

$$L_c > \frac{e}{r(1-\overline{p}_2)s_2} \tag{5c'}$$

$$L_c < \frac{\overline{p}_2 L_c' - e}{r(1+\overline{p}_2)(1-s_2)} \tag{5d'}$$

By arranging the above conditions, we can characterize the effect of L_c and L_c' on equilibrium outcome in Figure 4.

From Figure 4 we can see that if we can set both L_c and L_c' greatly, both kinds of auditors will be forced to work hard, and we can get the most effective noisy equilibrium outcome: Equilibrium 3. If L_c, is very small and L_c' is large enough the auditor is not afraid of being sued by the outsiders but he dare not offend the manager who reports N_2. Hence, shirk and endorse the manager becomes the optimal strategy and the least effective outcome- Equilibrium 2 occurs. If L_c is at median value, type I auditor still prefers to shirk while type II auditor prefers to work hard. And with a large L_c', Equilibrium 4 is attained. Finally, if L_c' is at median value, and L_c is large enough, it is safer for type I auditor to shirk and qualify the report, while type II auditor chooses to work hard. Thus, Equilibrium 5 occurs. It should be noticed that in other cases, no strategy of the auditor can be sustained given the manager always chooses M_2. Then there will be no noisy equilibrium at all. From the above analysis we know that raising the punishment on the manager or the auditor does not always lead to the most effective equilibrium outcome. On the one hand, if L_a is very large it is costly for the manager to be qualified by the auditor. The incentive to misreport is hence lowered. However, the auditor's incentive to shirk will then increase and upset the most effective equilibrium. That is, we have to set the value of punishment on the manager for being qualified properly to sustain the most effective equilibrium.

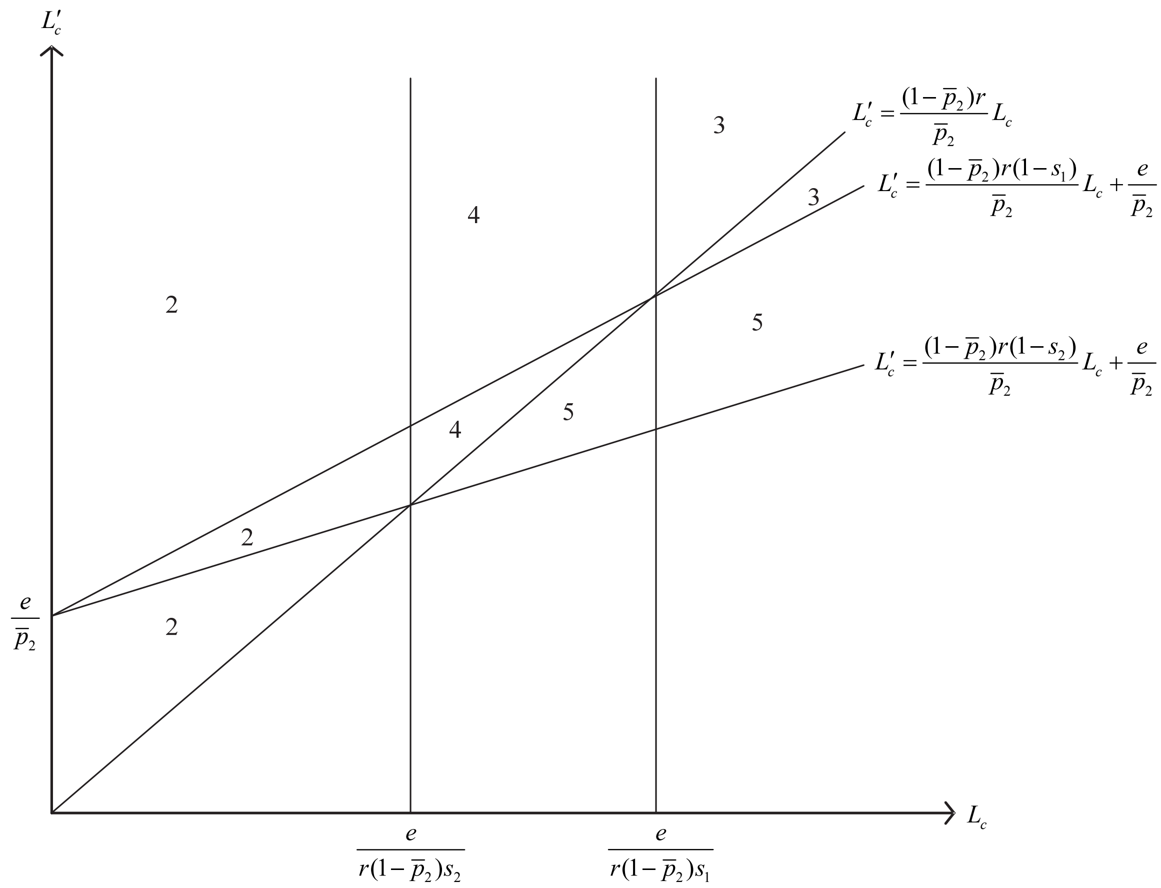

Figure 4 The equilibrium regions on different L_c and L_c'

On the other hand, with large L_c, the auditor will avoid being sued by the outsiders. And if L_c' is sufficiently small in the same time, the auditor would rather offend the manager by qualifying the financial report. But this might induce the manager to report the low net income even if the actual result is high. Then we still cannot get the most effective equilibrium, either. And we have already shown that in the case where L_c' is large while L_c is small, the least effective equilibrium occurs. Equilibrium 3 will be attained only when both L_c and L_c' are large.

It is clear that we can use Figures 2 and 3 to determine whether the punishment on the manager or the auditor is adequate. For example, if both kinds of punishment on the auditor, L_c and L_c' are large but we still observe that some auditors are shirking (that is, Equilibrium 4 occurs), we can then conclude that the punishment on the manager, L_a, is too severe. Thus, reducing L_a may be the right way to reach Equilibrium 3 and improve the quality of accounting information. That is, Figures 2 and 3 can characterize the relationship between actual state and punishments on the manager and the auditor. Then we can learn that what action should be taken to reach the desired outcome. It is interesting to investigate further about its effect on the social welfare if we change the magnitude of punishment to get the desired outcome. We can see that there are rich implications in policy making in this simple model.

5. Conclusions

We consider an incomplete information model of an auditing game to describe the interaction between the manager's and the auditor's strategies and to find out the possible equilibrium outcome in different conditions. Our contribution is two-fold. First, the penalties on managers' misreporting play a crucial role, which is neglected in most of previous research. Second, we put a lot of efforts on refinements to find out the equilibria that are reasonable and stable. Never an analytical research in accounting has done that before. The main conclusions we derive are: First, there exist many equilibria in our auditing game and the quality of accounting in those equilibria are quite different. Second, to simplify our analysis, we do a lot of refinements to find out those equilibria that are reasonable and stable. The conclusion is only the pure strategy equilibria can satisfy these requirements. Third, among these "good" equilibria, the noiseless equilibrium contains the highest quality of accounting information, but it is hard to attain. Therefore, it is interesting to investigate if the most effective noisy equilibrium can be reached through raising the punishment on the manager and/or the auditor. We find that this is not always true. On the one hand, if the punishment on the manager for being qualified is too heavy, the incentive for the auditor to shirk will increase and the most effective equilibrium cannot be reached. On the other hand, if the punishment on the auditor to be sued by the outsiders is very heavy, but the punishment for reporting the high net income as low is not heavy enough at the same time, the auditor would rather qualify the manager's financial report. And this might induce the manager to report the low net income even if the actual result is high. Thus, we cannot get the most effective equilibrium, either. Hence, the punishment on the manager and the auditor should be set properly to induce the most desired outcome. This result also has some implications in policy making: we can determine whether the punishments are adequate according to the strategies chosen by the manager and the auditor in the real world. Then we can learn to what extent the penalty should be imposed on the manager and the auditor according to some criterion, like social welfare. Thus, the analysis in this model is very helpful in policy making.

The model can be extended in various ways for future research. First, in this paper we assume the type of the manager or the auditor is private information. Thus, the manager cannot signal his own type by choosing the auditor and neither can the auditor. This assumption can be relaxed by allowing the manager or the auditor to send signals before they meet. For example, the auditor can offer management advisory service to build up his reputation. The manager can publish the earnings forecast in advance to show that he can predict any kind of change in the environment. Then a "matching process" can be added to our model. And we can study how the matching between managers and auditors is affected by the exogenous variables like the punishment in our model.

The above discussion can be applied to empirical work, too. For example, we can choose the size of the manager's firm and the size of the auditor's CPA firm as proxies for the manager's type and the auditor's type respectively. Then we can test the statistical relationship between these two proxies under various conditions. It might be claimed that the large firm's manager tends to match with the large CPA firm's auditor in some conditions and with the small CPA firm's auditor in other conditions. We can use the empirical result to verify the conclusion derived from the analysis of the model.

Another way of extension is to study the multi-period problem. That is, the manager and the auditor will play our one-shot game many times. After each round, the manager and the auditor can decide whether to maintain the relationship with each other in the next period. The main advantage of this modified story is that it characterizes the reputation problem more clearly than our model does. For example, if the auditor shirks and qualifies the financial report when the manager reports the high net income truthfully in one period, and the manager responds with replacing him in the next period. Then the auditor being replaced will be classified as the "bad" type and suffers the reputation loss. Thus the auditor's incentive to shirk will be less compared to our one period model.

Finally, in the real world if the auditor does not agree with the content in the financial report, he should communicate with the manager first. If the manager does not accept the auditor’s opinion, the auditor will then qualify the financial report. This scenario can be described by adding a “bargaining process” between manager and auditor in the model. In the bargaining process both manager and auditor have many strategies. For example, the manager’s strategy may be accepting the auditor’s opinion if the suggestion made by the auditor does not require too many corrections. It is clear that the model will be very complicated if we include such a process.

Appendix A: Summary of notations

N_1	Low net income.
N_2	High net income.
S_1	The probability for type I auditor to defect errors in the financial report provided by the manager.
S_2	The probability for type II auditor to defect errors in the financial report provided by the manager.
p	The probability for the manager to get N_1.
C_1	The auditor’s strategy: shirk and endorse the manager’s financial report.
C_2	The auditor’s strategy: shirk and qualify the manager’s financial report.
C_3	The auditor’s strategy: work hard and report truthfully.
M_1	The manager’s strategy: report N_1.
M_2	The manager’s strategy: report N_2.
α	The manager’s share of net income.
L_a	The punishment on the manager if the financial report is qualified by the auditor.
L_m	The punishment on the manager if the error in the financial report is found by the outsiders.
L_c	The punishment on the manager if he doesn’t find the error, which is found later by the outsiders.
L_c'	The punishment on the manager if he qualifies the financial report with high net income which is the truth.
r	The probability for the outsiders to find the errors in the financial report.
e	The auditor ‘s effort cost if he chooses to work hard.
F_2	The audit fee paid to the auditor if he endorses the financial report with high net income.
F_1	The audit fee paid to the auditor in other cases.
$\overline{p}_2 = p\left(N_2 \mid M_2\right)$	The auditor’s belief about the actual net income when the manager chooses M_2.
$\overline{p}_1 = p\left(N_1 \mid M_1\right)$	The auditor’s belief about the actual net income when the manager chooses M_1.
$\overline{s}$	The ex ante expected probability for the auditor to find errors in the financial report.
σ^m	The manager’s mixed strategy.
σ^c	The auditor’s mixed strategy.
t_{ij}	The probability for the manager to choose (M_i, M_j) in σ^m.
u_{ij}	The probability for the type I auditor to choose (C_i, C_j) in σ^c.
v_{ij}	The probability for the type II auditor to choose (C_i, C_j) in σ^c.
$VM(x \mid y)$	The manager’s expected payoff choosing x provided that the auditor chooses *y*.
$VC_{\mathrm{I}}(y \mid x)$	The type I auditor’s expected payoff choosing *y* provided that the manager chooses *x*.
$VC_{\mathrm{II}}(y \mid x)$	The type II auditor’s expected payoff choosing *x* provided that the manager chooses *x*.

Appendix B: Proofs

Proof of lemma 2

Suppose there is a noiseless equilibrium, denoted by *E*, in which the auditor might choose C_3. Because *E* is noiseless, the manager must tell the truth all the time. But then it is irrational for the auditor to work hard. Thus, *E* cannot be an equilibrium, a contradiction. Hence, the auditor chooses C_3 cannot be a part of a noiseless equilibrium. Q.E.D.

Proof of proposition 1

If $\alpha N_1 > \alpha N_2 - rL_m$, Equilibrium 1 can be sustained. In fact, in this case there exists no other equilibrium.[17] If $\alpha N_1 = \alpha N_2 - rL_m$, the manager will only randomize between M_1 and M_2 given the auditor chooses C_1 when the actual result is N_1, so noiseless equilibrium does not exist. If $\alpha N_1 < \alpha N_2 - rL_m$, it is possible for the manager to always tell the truth only if the auditor chooses C_3. But from lemma 2, auditor chooses C_3 cannot happen in a noiseless equilibrium. Thus, Equilibrium 1 is the only possible noiseless equilibrium. Q.E.D.

Proof of proposition 2

An equilibrium will be a sequential equilibrium if it satisfies sequential rationality and consistency. It can be easily shown that if the conditions stated in Equilibrium 3 are all satisfied, then starting from every information set, the strategies of Equilibrium 3 are optimal for the manager (or auditor) in that information set for the remainder of the game given that he is evaluating the payoffs according to his belief on the nodes in that information set and on his expectations of the other one's strategy. Thus, sequential rationality is satisfied.

To show that the equilibrium is consistent, we assume that both manager and auditor have some probability to tremble away from equilibrium strategy. Let $\{\sigma_n^m\}_{n=1}^{\infty}$ and $\{\sigma_n^c\}_{n=1}^{\infty}$ be the sequences of the manager's and the auditor's strategies respectively, where:

σ_n^m: chooses M_2 with probability $1-\frac{\varepsilon}{n}$ and M_1 with probability $\frac{\varepsilon}{n}$ when the actual net income is N_1;

chooses M_2 with probability $1-\frac{\varepsilon}{n^2}$ and M_1 with probability $\frac{\varepsilon}{n^2}$ when

[17] From lemma 1, in this case (M_1, M_2) is the manager's dominant strategy. Thus, the auditor's best response is (C_1, C_1).

the actual net income is N_2.

σ_n^c: chooses C_1 with probability $1-\frac{2\varepsilon}{n^2}$, C_2 with probability $\frac{\varepsilon}{n^2}$, and C_3 with probability $\frac{\varepsilon}{n^2}$ when the net income in the financial report is N_1;

chooses C_3 with probability $1-\frac{2\varepsilon}{n}$, C_1 with probability $\frac{\varepsilon}{n}$, and C_2 with probability $\frac{\varepsilon}{n}$ when the net income in the financial report is N_2.

And we let $\{\mu_n\}_{n=1}^{\infty}$ be the sequence of the auditor's belief when he receives the financial report, where:

μ_n: N_1 with probability $\dfrac{(1-\frac{\varepsilon}{n})\overline{p}_2}{(1-\frac{\varepsilon}{n})\overline{p}_2+(1-\frac{\varepsilon}{n^2})(1-\overline{p}_2)}$ and N_2 with probability $\dfrac{(1-\frac{\varepsilon}{n})(1-\overline{p}_2)}{(1-\frac{\varepsilon}{n})\overline{p}_2+(1-\frac{\varepsilon}{n^2})(1-\overline{p}_2)}$ when the net income in the financial report is N_2;

N_1 with probability $\dfrac{\frac{\varepsilon}{n}\overline{p}_1}{\frac{\varepsilon}{n}\overline{p}_1+\frac{\varepsilon}{n^2}(1-\overline{p}_1)}$ and N_2 with probability

$\dfrac{\frac{\varepsilon}{n^2}(1-\overline{p}_1)}{\frac{\varepsilon}{n}\overline{p}_1+\frac{\varepsilon}{n^2}(1-\overline{p}_1)}$ when the net income in the financial report is N_1.

We can easily check that μ_n are Bayes' consistent with σ_n^m and σ_n^c. And it is straightforward that:

$$\lim_{n\to\infty}(\sigma_n^m,\sigma_n^c,\mu_n)=(\sigma^m,\sigma^c,\mu)$$

where (σ^m,σ^c,μ) denotes the strategy profile and beliefs characterized in Equilibrium 3. Thus, Equilibrium 3 satisfies consistency too. And we have proved that Equilibrium 3 is a sequential equilibrium. Q.E.D.

<u>Proof of proposition 4</u>

We can find a sequence $\{\varepsilon_k\}_{k=1}^{\infty}$ in which $\varepsilon_k=\frac{1}{k}$. It is obvious that ε_k

converges to zero, condition (i) is satisfied. Then let $\{\sigma_k^m\}_{k=1}^{\infty}$ and $\{\sigma_k^c\}_{k=1}^{\infty}$ be the sequences of the manager's and the auditor's totally mixed strategies, where

σ_k^m: chooses M_1 with probability $1-\frac{1}{k+1}$ and M_2 with probability $\frac{1}{k+1}$ when the actual net income is N_1;

chooses M_2 with probability $1-\left(\frac{1}{k+1}\right)^n$ and M_1 with probability $\left(\frac{1}{k+1}\right)^n$ when the actual net income is N_2, where $n>1$, $n\in R$.

σ_k^c: chooses C_1 with probability $1-\frac{2}{(k+1)^2}$, C_2 with probability $\frac{1}{(k+1)^2}$, and C_3 with probability $\frac{1}{(k+1)^2}$ no matter what kind of financial report he receives.

To check if (σ_k^m,σ_k^c) is ε-perfect, first we look at the manager's side:

(a) When the actual net income is N_1, we can compute:

$$VM(M_1\mid\sigma_k^c)=(1-\frac{2}{(k+1)^2})\alpha N_1+\frac{1}{(k+1)^2}(\alpha N_2-L_a)+\frac{1}{(k+1)^2}\alpha N_1$$

$$VM(M_2\mid\sigma_k^c)=(1-\frac{2}{(k+1)^2})(\alpha N_2-rL_m)+\frac{1}{(k+1)^2}(\alpha N_2-L_a)$$
$$+\frac{1}{(k+1)^2}[\bar{s}\ (\alpha N_1-L_a)+(1-\bar{s}\ )(\alpha N_2-rL_m)]$$

Since we know that $\alpha N_1>\alpha N_2-rL_m$ under Equilibrium 1, we have $VM(M_1\mid\sigma_k^c)>VM(M_2\mid\sigma_k^c)$ when the actual net income is N_1.

(b) When the actual net income is N_2, we can compute:

$$VM(M_1\mid\sigma_k^c)=(1-\frac{2}{(k+1)^2})(\alpha N_1-rL_m)+\frac{1}{(k+1)^2}(\alpha N_1-rL_m)$$
$$+\frac{1}{(k+1)^2}[\bar{s}(\alpha N_2-L_a)+(1-\bar{s})(\alpha N_1-rL_m)]$$

$$VM(M_2\mid\sigma_k^c)=(1-\frac{2}{(k+1)^2})\alpha N_2+\frac{1}{(k+1)^2}(\alpha N_1-rL_m)+\frac{1}{(k+1)^2}\alpha N_2$$

And it is trivial to see that $VM(M_2\mid\sigma_k^c)>VM(M_1\mid\sigma_k^c)$ when the actual net income is N_2.

Hence, when the actual net income is N_1 (N_2), M_2 (M_1) is the dominated strategy for the manager. Let $\delta_m(M_i)$ denote the weight the manager puts on M_i in σ_k^m. From the definition of ε-perfect equilibrium, it has to be that $\delta_m(M_i) \le \varepsilon_k$ for all k if M_i is a dominated strategy for the manager. And it is clear that: $\delta_m(M_2) = \frac{1}{k+1} < \frac{1}{k} = \varepsilon_k$ when the actual net income is N_1 and $\delta_m(M_1) = (\frac{1}{k+1})^n < \frac{1}{k} = \varepsilon_k$ when the actual net income is N_2.

Thus, from the manager's side, (σ_k^m, σ_k^c) can satisfy the requirement to be an ε-perfect equilibrium.

And from the auditor's side:

(a) For type I auditor, when the net income in the financial report is N_1, we can compute the expected payoff of choosing C_1, C_2 and C_3:

$$VC_I(C_1 \mid \sigma_k^m) = \frac{(1-\frac{1}{k+1})\overline{p}_2}{(1-\frac{1}{k+1})\cdot\overline{p}_2 + (\frac{1}{k+1})^n(1-\overline{p}_2)} \cdot F_1 + \frac{(\frac{1}{k+1})^n(1-\overline{p}_2)}{(1-\frac{1}{k+1})\cdot\overline{p}_2 + (\frac{1}{k+1})^n(1-\overline{p}_2)} \cdot (F_1 - rL_c)$$

$$VC_I(C_2 \mid \sigma_k^m) = \frac{(1-\frac{1}{k+1})\overline{p}_2}{(1-\frac{1}{k+1})\cdot\overline{p}_2 + (\frac{1}{k+1})^n(1-\overline{p}_2)} \cdot (F_1 - rL_c) + \frac{(\frac{1}{k+1})^n(1-\overline{p}_2)}{(1-\frac{1}{k+1})\cdot\overline{p}_2 + (\frac{1}{k+1})^n(1-\overline{p}_2)} \cdot F_1$$

$$VC_I(C_3 \mid \sigma_k^m) = \frac{(1-\frac{1}{k+1})\overline{p}_2}{(1-\frac{1}{k+1})\cdot\overline{p}_2 + (\frac{1}{k+1})^n(1-\overline{p}_2)} \cdot (F_1 - e) + \frac{(\frac{1}{k+1})^n(1-\overline{p}_2)}{(1-\frac{1}{k+1})\cdot\overline{p}_2 + (\frac{1}{k+1})^n(1-\overline{p}_2)} \cdot \begin{bmatrix} s_1F_1 \\ +(1-s_1)(F_2 - rL_c) - e \end{bmatrix}$$

Apparently, we can choose n so that $VC_I(C_1 \mid \sigma_k^m) > VC_I(C_2 \mid \sigma_k^m)$ and $VC_I(C_1 \mid \sigma_k^m) > VC_I(C_3 \mid \sigma_k^m)$. That is, there exists a σ_k^m such that C_2 and C_3 are dominated by C_1 for the auditor when he receives a financial report with net income N_1.

(b) When the net income in the financial report is N_2, we can compute:

$$VC_I(C_1 \mid \sigma_k^m) = \frac{\frac{1}{k+1}\cdot \bar{p}_2}{\frac{1}{k+1}\cdot \bar{p}_2 + \left[1-(\frac{1}{k+1})^n\right](1-\bar{p}_2)}\cdot (F_2 - rL_c) + \frac{\left[1-(\frac{1}{k+1})^n\right](1-\bar{p}_2)}{\frac{1}{k+1}\cdot \bar{p}_2 + \left[1-(\frac{1}{k+1})^n\right](1-\bar{p}_2)}\cdot F_2;$$

$$VC_I(C_2 \mid \sigma_k^m) = \frac{\frac{1}{k+1}\cdot \bar{p}_2}{\frac{1}{k+1}\cdot \bar{p}_2 + \left[1-(\frac{1}{k+1})^n\right](1-\bar{p}_2)}\cdot F_1 + \frac{\left[1-(\frac{1}{k+1})^n\right](1-\bar{p}_2)}{\frac{1}{k+1}\cdot \bar{p}_2 + \left[1-(\frac{1}{k+1})^n\right](1-\bar{p}_2)}\cdot (F_1 - L_c');$$

$$VC_I(C_3 \mid \sigma_k^m) = \frac{\frac{1}{k+1}\cdot \bar{p}_2}{\frac{1}{k+1}\cdot \bar{p}_2 + \left[1-(\frac{1}{k+1})^n\right](1-\bar{p}_2)}\cdot \begin{bmatrix} s_1F_1 \\ +(1-s_1)(F_2 - rL_c) - e \end{bmatrix} + \frac{\left[1-(\frac{1}{k+1})^n\right](1-\bar{p}_2)}{\frac{1}{k+1}\cdot \bar{p}_2 + \left[1-(\frac{1}{k+1})^n\right](1-\bar{p}_2)}\cdot (F_2 - e).$$

Apparently, there exists a σ_k^m such that C_1 dominates C_2 and C_3 for the auditor when he receives a financial report with net income N_2. And we can easily derive the same conclusion for type II too. Now let $\delta_c(C_i)$ denotes the weight the auditor puts on C_i of δ_k^c. It is trivial to see that:

$\delta_c(C_3) = \dfrac{1}{(k+1)^2} < \dfrac{1}{k} = \varepsilon_k$ when the net income in financial report is N_1.

$\delta_c(C_2) = \delta_c(C_3) = \dfrac{1}{(k+1)^2} < \dfrac{1}{k} = \varepsilon_k$ when the net income in financial report is N_2.

Thus, from the auditor's side, (σ_k^m, σ_k^c) also satisfies the requirement of an ε-perfect equilibrium. Hence, condition (ii) is satisfied. Finally, it is easy to see that: $\lim_{k\to\infty}(\sigma_k^m, \sigma_k^c) = (\sigma^m, \sigma^c)$, where (σ^m, σ^c) is the strategy profile in Equilibrium 1. Thus, condition (iii) is satisfied too. And this completes the proof that Equilibrium 1 is trembling-hand perfect. Q.E.D.

Proof of proposition 5

For most of the mixed strategy profile (σ^m, σ^c) of mixed equilibria we can find $\{\varepsilon_k\}_{k=1}^{\infty}$, $\{\sigma_k^m\}_{k=1}^{\infty}$, and $\{\sigma_k^c\}_{k=1}^{\infty}$, in the similar way of the proof of proposition 4, to show that (σ_k^m, σ_k^c) is ε-perfect and converges to (σ^m, σ^c). But we cannot always do that. For example, consider the following mixed strategy profile and beliefs:

σ^m: chooses M_2 when the actual net income is N_2;
chooses M_1 with probability t and chooses M_2 with probability $1-t$ when the actual net income is N_1.

σ^c: chooses (C_1, C_1)

Manager's belief: P (type I auditor) = $\dfrac{1}{2}$

Auditor's belief: $P(N_2 \mid M_2) = \overline{p}_2$;
$P(N_1 \mid M_1) = 1$.

The conditions to sustain this equilibrium are:

$$F_2 - F_1 > (1-\overline{p}_2)rL_c - \overline{p}_2 L'_c$$

$$s_1(1-\overline{p}_2)rL_c - (1-\overline{p}_2)s_1(F_2 - F_1) < e$$

$$s_2(1-\overline{p}_2)rL_c - (1-\overline{p}_2)s_2(F_2 - F_1) < e$$

$$\alpha N_1 = \alpha N_2 - rL_m$$

where $\overline{p}_2 = \dfrac{(1-t)p}{(1-t)p + (1-p)}$

Now we arbitrarily select the sequences $\{\sigma_k^m\}_{k=1}^{\infty}$ and $\{\sigma_k^c\}_{k=1}^{\infty}$ as:

σ^m: chooses M_2 with probability $1-f_1(k)$ and M_1 with probability $f_1(k)$ when the actual net income is N_2;
chooses M_1 with probability $t-f_2(k)$ and M_2 with probability $1-t+f_2(k)$ when the actual net income is N_1.

σ^c : chooses C_1 with probability $1-f_3(k)-f_4(k)$ and C_2 with probability $f_3(k)$ and C_3 with probability $f_4(k)$ no matter what kind of financial report he receives,

where $0<f_1(k)<1$,

$0<t-f_2(k)<1$,

$0<1-t+f_2(k)<1$,

$0<f_3(k)<1$,

$0<f_4(k)<1$,

$f_3(k)+f_4(k)<1$.

Then, when the actual net income is N_1, we have:

$$VM(M_1 \mid \sigma_k^c) = [1-f_3(k)-f_4(k)]\cdot \alpha N_1 + f_3(k)\cdot(\alpha N_2 - L_a) + f_4(k)\cdot \alpha N_1;$$

$$VM(M_2 \mid \sigma_k^c) = [1-f_3(k)-f_4(k)]\cdot(\alpha N_2 - rL_m) + f_3(k)\cdot(\alpha N_1 - L_a) + f_4(k)\cdot[s_1(\alpha N_1 - L_a)+(1-s_1)(\alpha N_2 - rL_m)].$$

With $\alpha N_2 - rL_m = \alpha N_1 > \alpha N_1 - L_a$, it is straightforward that M_2 is dominated by M_1 for the manager when the actual net income is N_1. Thus, to sustain the above equilibrium to be trembling-hand perfect, the weight on M_2 must be very small. But this contradicts the equilibrium strategy. Hence, when $\alpha N_1 = \alpha N_2 - rL_m$, the mixed strategy equilibrium is not trembling-hand perfect.

Notice that in other cases, M_2 may not be dominated by M_1. For example, if $\alpha N_2 - rL_m > \alpha N_1 > \alpha N_1 - L_a$, we can always find the adequate $f_3(k)$ and $f_4(k)$ to make $VM(M_1 \mid \sigma_k^c)$ equal to $VM(M_2 \mid \sigma_k^c)$ and make the equilibria become trembling-hand perfect. Q.E.D.

<u>Proof of proposition 6</u>

For any small number $\varepsilon>0$, let $\delta=(\delta_1,\delta_2)$ with $0<\delta_i<\delta_0(\varepsilon)$ be a vector of positive constants smaller than δ_0. And let (π^m,π^c) be a vector of completely mixed strategy of the manager and the auditor in our original game, *G*, where:

π^m : chooses M_1 with probability W_3;

chooses M_2 with probability $1-W_3$.

π^c : chooses C_1 with probability W_1;

chooses C_2 with probability W_2;

chooses C_3 with probability $1-W_1-W_2$;

where $0 < W_i < 1$, $i = 1,2,3$.

Now we define the perturbed game, denoted by G', where the manager and the auditor are constrained to use completely mixed strategies as follows:

$\hat{M}_i = (1-\delta_1)M_i + \delta_1\pi^m$ for the manager $i = 1,2$;

$\hat{C}_j = (1-\delta_2)C_i + \delta_2\pi^c$ for the auditor $j = 1,2,3$.

Then consider the following strategy profile $(\hat{\sigma}^m, \hat{\sigma}^c)$ for the manager and the auditor in G':

$\hat{\sigma}^m$: chooses $(\hat{M}_1, \hat{M}_2)$;

$\hat{\sigma}^c$: chooses $(\hat{C}_1, \hat{C}_1)$.

And the consistent posterior beliefs, denoted by $\hat{\mu}$, for the auditor when he gets financial report with net income N_1 and N_2 can be computed via Bayes' rule as:

$\hat{u}$: N_1 with probability p' and N_2 with probability $1-p'$ when he gets the financial report with net income N_1;

N_1 with probability p'' and N_2 with probability $1-p''$ when he gets the financial report with net income N_2;

where
$$p' = \frac{\overline{p}_2\left[(1-\delta_1)+\delta_1 W_3\right]}{\overline{p}_2\left[(1-\delta_1)+\delta_1 W_3\right]+(1-\overline{p}_2)\delta_1 W},$$

$$p'' = \frac{\overline{p}_2\delta_1(1-W_3)}{\overline{p}_2\delta_1(1-W_3)(1-\overline{p}_2)\left[(1-\delta_1)+\delta_1(1-W_3)\right]}.$$

Next we want to check if the strategy profile and belief $(\hat{\sigma}^m, \hat{\sigma}^c, \hat{u})$ can form an equilibrium in G'. First from the manager's side, when the actual net income is N_1, we have:

$$\begin{aligned}
&VM(\hat{M}_1 \mid \hat{\sigma}^c)\\
&= \left[(1-\delta_1)+\delta_1 W_3\right]\{(1-\delta_2)\alpha N_1 + \delta_2[W_1\alpha N_1 + (1-W_1-W_2)\alpha N_1]\}\\
&+\left[(1-\delta_1)+\delta_1 W_3\right]\delta_2 W_2(\alpha N_2 - L_a) + \delta_1(1-W_3)(1-\delta_2)(\alpha N_2 - rL_m)\\
&+\delta_1(1-W_3)\delta_2\begin{pmatrix} W_1(\alpha N_2 - rL_m) + W_2(\alpha N_1 - L_a)\\ +(1-W_1-W_2)\left[s_1(\alpha N_1 - L_a)+(1-s_1)(\alpha N_2 - rL_m)\right]\end{pmatrix};
\end{aligned}$$

$$VM(\hat{M}_2 \mid \hat{\sigma}^c)$$
$$= (1-\delta_1)(1-\delta_2)(\alpha N_2 - rL_m)$$
$$+(1-\delta_1)\delta_2 \begin{pmatrix} (W_1(\alpha N_2 - rL_m) + W_2(\alpha N_1 - L_a) \\ +(1-W_1-W_2)\left[s_1(\alpha N_1 - L_a) + (1-s_1)(\alpha N_2 - rL_m)\right] \end{pmatrix}$$
$$+\delta_1 W_3\{(1-\delta_2)\alpha N_1 + \delta_2[W_1\alpha N_1 + W_2\alpha N_1 + (1-W_1-W_2)\alpha N_1]\}$$
$$+\delta_1(1-W_3)\delta_2 \begin{pmatrix} W_1(\alpha N_2 - rL_m) + W_2(\alpha N_1 - L_a) \\ +(1-W_1-W_2)\left[s_1(\alpha N_1 - L_a) + (1-s_1)(\alpha N_2 - rL_m)\right] \end{pmatrix}.$$

Because $\alpha N_1 > \alpha N_2 - rL_m$ and δ_1, δ_2 are very small numbers, we have $VM(\hat{M}_1 \mid \hat{\sigma}^c) > VM(\hat{M}_2 \mid \hat{\sigma}^c)$. Hence, when the actual net income is N_1, the best response for the manager in the perturbed game is $\hat{M}_1$. And following the same algorithm we can also show that $\hat{M}_2$ is the best response when the actual net income is N_2. That is, $\hat{\sigma}^m$ is the best response for the manager against $\hat{\sigma}^c$ in the perturbed game.

From the auditor's side, when a type I auditor receives the financial report with net income N_1, we can compute:

$$VC_I(\hat{C}_1 \mid \hat{\sigma}^m)$$
$$= \left[(1-\delta_2) + \delta_2 W_1\right]\left[p'F_1 + (1-p')(F_1 - rL_a)\right] + \delta_2 W_2\left[p'(F_1 - rL_c) + (1-p')F_1\right]$$
$$+\delta_1(1-W_1-W_2)\{p'(F_1 - e) + (1-p')\left[s_1 F_1 + (1-s_1)(F_1 - rL_c) - e\right]\};$$
$$VC_I(\hat{C}_2 \mid \hat{\sigma}^m)$$
$$= \left[(1-\delta_2) + \delta_2 W_2\right]\left[p'(F_1 - rL_c) + (1-p')F_1\right] + \delta_2 W_1\left[p'F_1 + (1-p')(F_1 - rL_c)\right]$$
$$+\delta_1(1-W_1-W_2)\{p'(F_1 - e) + (1-p')\left[s_1 F_1 + (1-s_1)(F_1 - rL_c) - e\right]\};$$
$$VC_I(\hat{C}_3 \mid \hat{\sigma}^m)$$
$$= \left[(1-\delta_2) + \delta(1-W_1-W_2)\right]\{p'(F_1 - e) + (1-p')\left[s_1 F_2 + (1-s_1)(F_1 - rL_c) - e\right]\}$$
$$+\delta_2 W_1\left[p'F_1 + (1-p')(F_1 - rL_c)\right] + \delta_2 W_2\left[p'(F_1 - rL_c) + (1-p')F_1\right].$$

p' is very close to 1 due to δ_2 is very small. Thus, it is clear that $VC_I(\hat{C}_1 \mid \hat{\sigma}^m) > VC_I(\hat{C}_2 \mid \hat{\sigma}^m)$, $VC_I(\hat{C}_1 \mid \hat{\sigma}^m) > VC_I(\hat{C}_3 \mid \hat{\sigma}^m)$ and $VC_{II}(\hat{C}_1 \mid \hat{\sigma}^m) > VC_{II}(\hat{C}_2 \mid \hat{\sigma}^m)$, $VC_{II}(\hat{C}_1 \mid \hat{\sigma}^m) > VC_{II}(\hat{C}_3 \mid \hat{\sigma}^m)$ when the auditor receives a financial report with net income N_1 and in this case $\hat{C}_1$ and $\hat{C}_2$ are the best responses for the auditor against $\hat{\sigma}^m$. With the same way we can derive that $\hat{C}_1$ is the best response for the auditor when the net income in the financial report is

N_2 . And we have shown that $(\hat{\sigma}^m, \hat{\sigma}^c)$ is an equilibrium in the perturbed game G' .

Next we want to show that $(\hat{\sigma}^m, \hat{\sigma}^c)$ and (σ^m, σ^c), denotes the strategy profile in Equilibrium 1, are very close. This is done by computing the Euclidian distance between $(\hat{M}_1, \hat{M}_2, \hat{C}_1, \hat{C}_1)$ and (M_1, M_2, C_1, C_1) on R^4:

$$\text{dist}\left[(\hat{M}_1, \hat{M}_2, \hat{C}_1, \hat{C}_1), (M_1, M_2, C_1, C_1)\right]$$

$$= \sqrt{2(\delta_1 - \delta_1 W_3)^2 + 2(\delta_1 W_3)^2 + 2\left[(-\delta_2 + \delta_2 W_1)^2 + (\delta_2 W_2)^2 + \delta_2^2 (1 - W_1 - W_2)^2\right]}$$

$$< \sqrt{2}\delta_0 \sqrt{(1 - W_3)^2 + W_3^2 + (1 - W_1)^2 + W_2^2 + (1 - W_1 - W_2)^2}.$$

We can choose $\delta_0 = \dfrac{\varepsilon}{\sqrt{2} \cdot \sqrt{(1 - W_3)^2 + W_3^2 + (1 - W_1)^2 + W_2^2 + (1 - W_1 - W_2)^2}}$ such that $\text{dist}\left[(\hat{M}_1, \hat{M}_2, \hat{C}_1, \hat{C}_1), (M_1, M_2, C_1, C_1)\right] < \varepsilon$. That is, $(\hat{\sigma}^m, \hat{\sigma}^c)$ is ε -close to (σ^m, σ^c) . Hence, Equilibrium 1 satisfies property *S*. And it is naturally minimal with property *S* due to it's a one-point set. This completes the proof that Equilibrium 1 is a stable equilibrium. Q.E.D.

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